

Surds

1.

Simplify

a $\sqrt{18} + \sqrt{50}$

b $\sqrt{48} - \sqrt{27}$

c $2\sqrt{8} + \sqrt{72}$

d $\sqrt{360} - 2\sqrt{40}$

e $2\sqrt{5} - \sqrt{45} + 3\sqrt{20}$

f $\sqrt{24} + \sqrt{150} - 2\sqrt{96}$

2.

Express in the form $a + b\sqrt{3}$

a $\sqrt{3}(2 + \sqrt{3})$

b $4 - \sqrt{3} - 2(1 - \sqrt{3})$

c $(1 + \sqrt{3})(2 + \sqrt{3})$

d $(4 + \sqrt{3})(1 + 2\sqrt{3})$

e $(3\sqrt{3} - 4)^2$

f $(3\sqrt{3} + 1)(2 - 5\sqrt{3})$

3.

Simplify

a $\sqrt{8} + \frac{6}{\sqrt{2}}$

b $\sqrt{48} - \frac{10}{\sqrt{3}}$

c $\frac{6 - \sqrt{8}}{\sqrt{2}}$

d $\frac{\sqrt{45} - 5}{\sqrt{20}}$

e $\frac{1}{\sqrt{18}} + \frac{1}{\sqrt{32}}$

f $\frac{2}{\sqrt{3}} - \frac{\sqrt{6}}{\sqrt{72}}$

4.

Solve each equation, giving your answers as simply as possible in terms of surds.

a $x(x + 4) = 4(x + 8)$

b $x - \sqrt{48} = 2\sqrt{3} - 2x$

c $x\sqrt{18} - 4 = \sqrt{8}$

d $x\sqrt{5} + 2 = \sqrt{20}(x - 1)$

5.

Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{2} + 1}$

b $\frac{4}{\sqrt{3} - 1}$

c $\frac{1}{\sqrt{6} - 2}$

d $\frac{3}{2 + \sqrt{3}}$

e $\frac{1}{2 + \sqrt{5}}$

f $\frac{\sqrt{2}}{\sqrt{2} - 1}$

g $\frac{6}{\sqrt{7} + 3}$

h $\frac{1}{3 + 2\sqrt{2}}$

i $\frac{1}{4 - 2\sqrt{3}}$

j $\frac{3}{3\sqrt{2} + 4}$

k $\frac{2\sqrt{3}}{7 - 4\sqrt{3}}$

l $\frac{6}{\sqrt{5} - \sqrt{3}}$

Surds Problem Solving (Ex 1.3B Q 3, 7, 9, 14, 15)

1.

a A car travels $18\sqrt{35}$ m in $6\sqrt{7}$ s. Work out its speed, showing your working.

b A runner travels for 5 s at $\frac{8}{\sqrt{5}}$ m s⁻¹.

Work out how far she ran in simplified form. Show your working.

2.

Base camp is $5\sqrt{5}$ miles due east and $5\sqrt{7}$ miles due north of a walker. What is the exact distance from the walker to the camp? Show your working.

3.

The equation of a parabola is $y^2 = 4ax$

Find y when $a = 6 - \sqrt{6}$ and $x = \frac{6 + \sqrt{6}}{10}$

Show your working and give your answer in simplified form.

4.

An equilateral triangle with side length $5\sqrt{6}$ inches has one vertex at the origin and one side along the positive x -axis.

The centre is on the vertical line of symmetry,

$\frac{1}{3}$ of the way from the x -axis to the vertex.

Work out the distance from the origin to the centre of mass of the triangle. Show your working.

5.

The Indian Mathematician

Brahmagupta (598 – 670) developed a formula to calculate the area of a cyclic quadrilateral.

If the sides of the quadrilateral are a , b ,

c and d , and $s = \frac{a+b+c+d}{2}$, the

area is $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$

A quadrilateral with side lengths

$5 + 5\sqrt{2}$, $3 + 3\sqrt{2}$, $6 + 4\sqrt{2}$ and

$2 + 4\sqrt{2}$ cm is inscribed in a circle.

Prove that $A = 2(1 + \sqrt{2})\sqrt{15(11 + 8\sqrt{2})}$

Surds Solutions

1.

$$a = 3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$$

$$b = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$$

$$c = 4\sqrt{2} + 6\sqrt{2} = 10\sqrt{2}$$

$$d = 6\sqrt{10} - 4\sqrt{10} = 2\sqrt{10}$$

$$e = 2\sqrt{5} - 3\sqrt{5} + 6\sqrt{5} = 5\sqrt{5}$$

$$f = 2\sqrt{6} + 5\sqrt{6} - 8\sqrt{6} = -\sqrt{6}$$

2.

$$a = 3 + 2\sqrt{3}$$

$$b = 4 - \sqrt{3} - 2 + 2\sqrt{3} \\ = 2 + \sqrt{3}$$

$$c = 2 + \sqrt{3} + 2\sqrt{3} + 3 \\ = 5 + 3\sqrt{3}$$

$$d = 4 + 8\sqrt{3} + \sqrt{3} + 6 \\ = 10 + 9\sqrt{3}$$

$$e = 27 - 24\sqrt{3} + 16 \\ = 43 - 24\sqrt{3}$$

$$f = 6\sqrt{3} - 45 + 2 - 5\sqrt{3} \\ = -43 + \sqrt{3}$$

3.

$$a = 2\sqrt{2} + \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ = 2\sqrt{2} + 3\sqrt{2} \\ = 5\sqrt{2}$$

$$b = 4\sqrt{3} - \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ = 4\sqrt{3} - \frac{10}{3}\sqrt{3} \\ = \frac{2}{3}\sqrt{3}$$

$$c = \frac{6-2\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ = \frac{6\sqrt{2}-4}{2} \\ = 3\sqrt{2} - 2$$

$$d = \frac{3\sqrt{5}-5}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ = \frac{15-5\sqrt{5}}{10} \\ = \frac{1}{2}(3 - \sqrt{5})$$

$$e = \frac{1}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ = \frac{1}{6}\sqrt{2} + \frac{1}{8}\sqrt{2} \\ = \frac{7}{24}\sqrt{2}$$

$$f = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{\sqrt{2}\sqrt{3}}{6\sqrt{2}} \\ = \frac{2}{3}\sqrt{3} - \frac{1}{6}\sqrt{3} \\ = \frac{1}{2}\sqrt{3}$$

4.

$$a \quad x^2 + 4x = 4x + 32 \\ x^2 = 32 \\ x = \pm\sqrt{32} \\ x = \pm 4\sqrt{2}$$

$$b \quad x - 4\sqrt{3} = 2\sqrt{3} - 2x \\ 3x = 6\sqrt{3} \\ x = 2\sqrt{3}$$

$$c \quad 3\sqrt{2}x - 4 = 2\sqrt{2} \\ 6x - 4\sqrt{2} = 4 \\ 6x = 4 + 4\sqrt{2} \\ x = \frac{2}{3}(1 + \sqrt{2})$$

$$d \quad \sqrt{5}x + 2 = 2\sqrt{5}(x - 1) \\ 5x + 2\sqrt{5} = 10(x - 1) \\ 5x = 10 + 2\sqrt{5} \\ x = 2 + \frac{2}{5}\sqrt{5}$$

5.

$$\mathbf{a} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

$$\mathbf{b} = \frac{4}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4(\sqrt{3}+1)}{3-1} = 2(\sqrt{3}+1)$$

$$\mathbf{c} = \frac{1}{\sqrt{6}-2} \times \frac{\sqrt{6}+2}{\sqrt{6}+2} = \frac{\sqrt{6}+2}{6-4} = \frac{1}{2}(\sqrt{6}+2) \text{ or } \frac{1}{2}\sqrt{6}+1$$

$$\mathbf{d} = \frac{3}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{3(2-\sqrt{3})}{4-3} = 3(2-\sqrt{3})$$

$$\mathbf{e} = \frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{2-\sqrt{5}}{4-5} = \sqrt{5}-2$$

$$\mathbf{f} = \frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}+1)}{2-1} = 2+\sqrt{2}$$

$$\mathbf{g} = \frac{6}{\sqrt{7}+3} \times \frac{\sqrt{7}-3}{\sqrt{7}-3} = \frac{6(\sqrt{7}-3)}{7-9} = 3(3-\sqrt{7})$$

$$\mathbf{h} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$$

$$\mathbf{i} = \frac{1}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}} = \frac{4+2\sqrt{3}}{16-12} = \frac{1}{2}(2+\sqrt{3}) \text{ or } 1+\frac{1}{2}\sqrt{3}$$

$$\mathbf{j} = \frac{3}{3\sqrt{2}+4} \times \frac{3\sqrt{2}-4}{3\sqrt{2}-4} = \frac{3(3\sqrt{2}-4)}{18-16} = \frac{3}{2}(3\sqrt{2}-4) \text{ or } \frac{9}{2}\sqrt{2}-6$$

$$\mathbf{k} = \frac{2\sqrt{3}}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{2\sqrt{3}(7+4\sqrt{3})}{49-48} = 2(7\sqrt{3}+12)$$

$$\mathbf{l} = \frac{6}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{6(\sqrt{5}+\sqrt{3})}{5-3} = 3(\sqrt{5}+\sqrt{3})$$

Surds Problem Solving Solutions

1.

a $3\sqrt{5} \text{ m s}^{-1}$ **b** $8\sqrt{5} \text{ m}$

2.

$$10\sqrt{3} \text{ m}$$

3.

$$2\sqrt{3}$$

4.

13

Thus coordinates of the centroid are $\left(\frac{5\sqrt{6}}{2}, \frac{5\sqrt{2}}{2}\right)$, so the distance from the origin to the centroid $= 5\sqrt{2}$

5.

$$2s = 5 + 5\sqrt{2} + 3 + 3\sqrt{2} + 6 + 4\sqrt{2} + 2 + 4\sqrt{2} = 16 + 16\sqrt{2}$$

$$s = 8 + 8\sqrt{2}$$

$$\begin{aligned} A^2 &= (3 + 3\sqrt{2})(5 + 5\sqrt{2})(2 + 4\sqrt{2})(6 + 4\sqrt{2}) \\ &= 3(1 + \sqrt{2})5(1 + \sqrt{2})2(1 + 2\sqrt{2})2(3 + 2\sqrt{2}) \\ &= 2^2(1 + \sqrt{2})^2 15(11 + 8\sqrt{2}) \end{aligned}$$

$$A = 2(1 + \sqrt{2})\sqrt{15(11 + 8\sqrt{2})}$$

(Full worked solutions on Kerboodle Ex1.3B Q 3,7,9,14,15)

